

# On the Coefficients of Quasiconformality for Convex Functions

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## Abstract

Let  $f$  be holomorphic and univalent in the unit disc  $E$  and  $f(E)$  be convex. We consider the conformal radius  $R = R(D, z) = \{ \rho(D, z) \} / \{ \rho(D, 0) \}$  of  $D = f(E)$  at the point  $z = f(\zeta)$ . In [3] and [4] the coefficient  $k_f(r)$ ,  $r \in (0, 1)$ , of quasiconformality has been defined by the equation, In this paper the authors computed the quantity  $k_f(r)$  for some convex functions. These examples led them to the conjecture that  $k_f(r) \leq r^2$  for any convex function holomorphic in  $E$ . The function  $f(\zeta) = \log((1 + \zeta)/(1 - \zeta))$ , which was among their examples, shows that this bound is sharp for any  $r \in (0, 1)$ . In the present article, we will prove that the above conjecture is true and that the above example is essentially the only one for which equality is attained. © 2010 Pleiades Publishing, Ltd.

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## Keywords

Coefficient of quasiconformality, Conformal radius, Convex functions